Common Factor Analysis: A Research Method for Understanding Dimensions of Ideas

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Shim, Jae-woo. 2002. Common Factor Analysis: A Research Method for Understanding Dimensions of Ideas. *The Linguistic Association of Korea Journal. 10*(2), 161-181. The purpose of this paper is to introduce a statistical technique of common factor analysis, which is often used for identifying underlying dimensions of ideas. To introduce this technique authentically, the researcher conducted a study to investigate dimensions of learner motivation among students of English as a foreign language. The subjects were 115 college students in Korea. The study found that the subjects held two dimensions of motivation, integrative motivation and instrumental motivation. The study also found that male subjects had higher integrative motivation than female subjects. The result corroborated other studies on learner motivation. Along with this study, the concepts of common factor analysis were explained. They include a definition of common factor analysis, statistics for checking the appropriateness of common factor analysis such as partial correlation coefficients, Kaiser-Meyer-Oklin measure of sampling adequacy (MSA), and Barlett's test of sphericity, eigen value specification, scree test, computation of the percentage of variance extracted, rotation of factors, interpretation of factors, use of factor scores for further analyses.

Key words: Research method, common factor analysis, and learner motivation

1. Introduction

Researchers in the field of teaching English as a second language or foreign language have begun to employ highly sophisticated statistical methods such as factor analysis, cluster analysis, multiple regression analysis, logistic regression analysis, multiple analysis of variance, discriminant analysis, and canonical analysis, to name a few.

Considering that research in language learning often involves understanding learners' perceptions, factor analysis has become more important than ever before.

Factor analysis is a multivariate statistical method applied to understand dimensions of ideas or common factors. For example, Park (2002) applied principal component factor analysis in developing an instrument that is intended to measure learners' English learning strategies. One of factors Park found was 'use of various materials for studying English.' The factor consisted of the following ideas: watching English study programs on TV, listening to English tapes, solving quizzes from daily English study materials, watching video programs on English, reading English cartoons, and using computers to learn English. By using a statistical technique, the author was able to group the ideas under the factor called 'use of various materials for studying English.' Also, Shim (2001) used principal component factor analysis to identify dimensions of teacher efficacy beliefs. For example, in the study, he found two dimensions of teacher efficacy, 'personal teaching efficacy' and 'general teaching efficacy.' According to the study, 'personal teaching efficacy' was derived from such items as trying hard, teacher know-how, class management knowledge, assessing difficulty, and confidence in effectiveness. Although principal component factor analysis is slightly different from common factor analysis, it has almost the same purpose as common factor analysis does, which is to reduce a number of variables or items to a less number of components or factors. However, common factor analysis has an advantage over principal component factor analysis in that it can find a factor solution which best fits observed correlations among variables in observed variable set.

The following sections of this paper will be focused on how to conduct common factor analysis and interpreting statistical results of it, using a set of data on learners' motivation.

2. A Pilot Study Designed For Explaining Common Factor Analysis

As stated earlier, common factor analysis is used to investigate what items (or variables) are grouped together to represent an idea which is also called a factor or a dimension. To show step-by-step procedures in conducting a factor analytic study, the researcher conducted a study. The researcher was interested in understanding dimensions of motivation of learners of English as a foreign language. The variable, language motivation has been widely investigated among second language researchers. Gardner(1989) suggested that there might be two types of motivation to learn a second language, instrumental motivation and integrative motivation. In literature, instrumental motivation is referred to as learners′ orientation to language learning that is activated when learners perceive social or economical advantages as a result of learning a particular language. Integrative motivation is referred to as learners′ orientation to know the culture of the target language and communicate with the members of the target group. Integrative motivation is usually grounded in positive attitude toward the L2 community. However, few studies were conducted to understand dimensions of learner motivation in the context of learning English as a foreign language, when the identification of learner motivation is critical in teaching and learning of English.

Oxford & Ehrman (1993) pointed out that gender received scant attention in research; few researchers have studied any motivational difference between male and female language learners. Research studies on any difference in motivation between males and females would help us understand better individual differences learners hold in learning English.

2.1. Research Questions

There are two research questions in the present study: 1) What are the dimensions (factors) of learner motivation among college students? 2) Is there any statistical difference in derived factor scores between male subjects and female subjects?

2.2. Subjects

Subjects of the present study were 115 college students. Subjects were taking English I classes when they were surveyed. Three classes were selected at random from a list of classes of the college the subjects were attending. A total of 140 students were asked to answer the questionnaire that contained items about their motives to learn English. They answered the questionnaire during their recess. Sixty seven male students and 48 female students returned the questionnaire. The response rate was 81%.

2.3. Materials and Procedures

Subjects were provided with Likert type scale questionnaire (Table 1) and asked to mark their ideas. The options ranged from *strongly disagree, disagree, slightly disagree, slightly agree, agree,* to *strongly agree.* Answers to *strongly disagree* received a score of 1, answers to *disagree* 2, answers to *slightly disagree* 3, answers to *slightly agree* 4, answers to *agree* 5, and answers to *strongly agree* a maximum score of 6.

The items of the present study were adapted by the researcher based on the items written by Clement, Dornyei, and Noels (1994). The items by Clement, Dornyei, and Noels contained ideas associated with two types of motivation Gardner hypothesized. The reliability of the instrument was .79 (Cronbach alpha). The research questionnaire for this study contained only 8 items which represented typical learner motivation. It is suggested that sample size be determined as a function of the number of variables being analyzed. Since the minimum ratio is 10 individuals to every variable or item, one would need 100 subjects if he or she has 10 items. Accordingly, this study with 8 items was better than having 10 items in terms of the ratio of subjects to items.

Table 1. Items of Motivation

Item No	Item Contents
1	Studying English will allow me to meet and converse with people from other nations.
2	Studying English is important for me because it will enable me to better understand American culture.
3	I study English because it will someday be useful in getting a job.
4	I study English because other people will respect me more if I have a good knowledge of it.
5	Studying English is important for me because I may need it for my career.
6	Knowing English will help me understand things better.
7	The more I learn English, the more I want to know Americans.
8	I will be more useful to society if I know English.

2.4. Findings

The present study found that the subjects held two kinds of motivations, integrative motivation and instrumental motivation. Also, it was revealed that male subjects had stronger integrative motivation than female subjects.

3. Processes of conducting common factor analysis

The processes of identifying factors include checking descriptive statistics, correlations of items, the appropriateness of applying factor analysis, and variance explained, rotating factors to have more clear factor solution, naming of factors, and if necessary, conducting further analyses based on derived factors. Descriptive statistics (Table 2) were checked.

Table 2. Descriptive Statistics

Item No	Mean	Std. Deviation	Analysis Number
1	2.8	1.34	115
2	2.9	1.27	115
3	3.7	1.38	115
4	3.8	1.34	115
5	4.5	1.02	115
6	3.3	1.13	115
7	2.8	1.25	115
8	2.9	1.22	115

Item 5 had the highest mean score, 4.5, while items 1 and 7 had the lowest. 2.8, respectively. Standard deviation ranged from 1.02 to 1.38. There is no particular item that should be concerned about because the variability of each item was low. It is also important to check correlations of items (Table 3).

Table 3. Correlations

Item No	1	2	3	4	5	6	7	8
1	1.0	.53	.38	.31	.10	.44	.57	.39
2	.53	1.0	.31	.26	.20	.31	.59	.29
3	.38	.31	1.0	.30	.10	.36	.34	.28
4	.31	.26	.30	1.0	.31	.39	.19	.32
5	.10	.20	.10	.31	1.0	.28	.24	.07
6	.44	.31	.36	.39	.28	.10	.42	.40
7	.57	.59	.34	.19	.24	.42	.10	.30
8	.39	.29	.28	.32	.07	.40	.30	1.0

The correlation table shows that some items have sizable correlations indicating the possibility of using factor analysis. For example, one group of items, that is, item 1, item 2, item 7 are correlated to each other, while the other group of items, that is, item 4, item 6, and item 8 are correlated to each other as well.

To decide if the factor analysis model is appropriate for the given data, the following tests are conducted: partial correlation coefficients, Barlett's test of sphericity, coefficients, and Kaiser-Meyer-Oklin measure of sampling adequacy (MSA). Table 4 shows anti-image matrices.

Table 4. Anti-Image Matrix

Item No	1	2	3	4	5	6	7	8
1	.83	-.25	-.12	-.11	.14	-.16	-.29	-.14
2	-.25	.81	0	0	0	0	-.38	0
3	-.12	0	.89	-.15	0	-.14	0	0
4	-.11	0	-.15	.77	-.26	-.17	.15	-.16
5	.14	0	0	-.26	.65	-.17	-.15	0
6	-.16	0	-.14	-.17	-.17	.84	-.16	-.21
7	-.29	-.38	0	.15	-.15	-.16	.78	0
8	-.14	0	0	-.16	0	-.21	0	.86

This anti-image matrix represents the negative value of the partial correlation. Partial correlation coefficients between pairs of items should be small when the linear effects of other variables have been eliminated. So, the smaller the partial correlation coefficients are, the more appropriate using common factor analysis is.

Kaiser-Meyer-Oklin measure of sampling adequacy (Table 5) is an index for comparing the magnitude of the observed correlation coefficients to the magnitude of the partial correlation coefficients. Factor analysis may not be appropriate if KMO statistic is low. In the present study, KMO value was .81, indicating the appropriateness of factor analysis for the study. Values of KMO range from 0 to 1. The value of KMO is near 1.0 if the sum of the squared partial correlation coefficients is small when compared to the sum of the squared correlation coefficients. Accordingly the small values for the KMO statistic indicate that factor analysis may not be appropriate. Interpreting the KMO statistic as a measure of sampling adequacy (MSA) for the correlation matrix and MSA values for individual variables, the following criteria are used (Hair, Anderson, Tatham & Black, pp. 99-100): .80 or above, meritorious, .70-.79, middling, .60-.69, mediocre, .50-.59, miserable, and below .50, unacceptable. Precisely, the formula for calculating KMO is as follows: KMO= sum of squared correlation coefficients between all pairs divided by sum of squared correlation coefficients between all pairs plus sum of squared partial correlation coefficients.

Table 5. KMO and Bartlett's Test

KMO Measure of Sampling Adequacy	Bartletts Test of Sphericity		
	Approx. Chi-Square	df	sig
.82	240.491	28	.000

Also, Bartletts test of sphericity was rejected in the present study as shown in Table 5, supporting the appropriateness for factor analysis. It is used to test the hypothesis that the correlation matrix is an identity matrix. An assumption of the test is that the data are a sample from a multivariate normal distribution and if "Ho: population correlation matrix is an identity matrix" cannot be rejected, the use of factor analysis should not be used. Accordingly, the rejection indicates that population correlation matrix contains correlations among items.

In Table 6, initial communality represents the proportion of variance explained in each item by the linear combinations of other items. Extraction communality refers to the proportion of variance explained in each item by the linear combinations of whatever factors extracted. For example, initial communality in item 1 is .47 and the number indicates that 47% of variance in item 1 is explained by all other items. Extraction communality for item 1 was .54 and it indicates that 54% of variance in item 1 is explained by factors. Communality is another indication of the strength of the association.

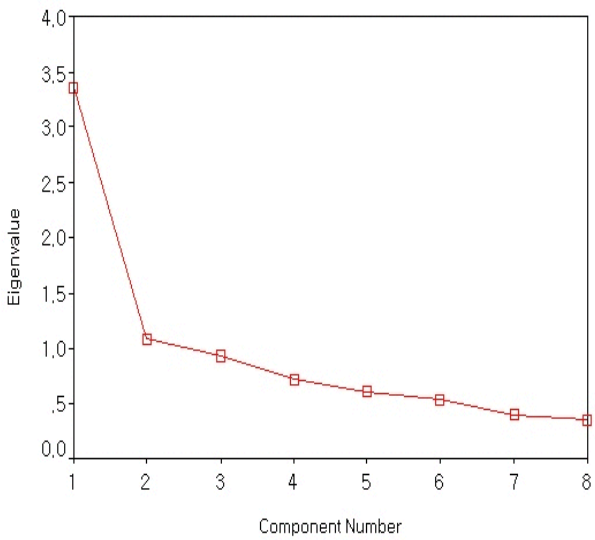
For extracting common factors in this study, maximum likelihood method was employed for this study. Other extracting methods include principal axis factoring and alpha method. With a maximum likelihood method, one can find the factor solution which best fits the observed correlations among variables in the observed variable set.

Table 6. Communalities

Item No	Initial	Extraction
1	.477	.548
2	.430	.514
3	.237	.277
4	.282	.473
5	.181	.126
6	.375	.456
7	.492	.684
8	.257	.295

In deciding the number of factors to retain in the model, a scree test is used (Table 7). Also, one may retain factors with eigenvalues equal to or greater than 1. The researcher retained factors with eigen values in the sharp descent before the point where the eigenvalues begin to level off forming a straight line. The researcher decided to extract 2 factors.

Table 7. Scree Plot



For the present study, about 42% of variance was explained by the extraction (Table 8). The first factor explained 23.3% of variance and the second factor explained 18.8% of variance. The percentage of total variance extracted is computed by dividing the eigenvalues for the factors extracted by the sum of the variances for all variables in the observed variable set.

Table 8. Total Variance Explained

Factor	Intial Eigenvalues			Extraction Sums of Sqr. Loadings			Rotation Sums of Sqr. Loadings		
	Total	% of Variance	Cum %	Total	% of Variance	Cum %	Total	% of Variance	Cum %
1	3.364	42.051	42.051	2.820	35.244	35.244	1.867	23.342	23.342
2	1.082	13.529	55.580	.554	6.920	42.164	1.506	18.822	42.164
3	.936	11.703	67.284						
4	.713	8.917	76.201						
5	.615	7.682	83.883						
6	.534	6.669	90.552						
7	.401	5.018	95.570						
8	.354	4.430	100.000						

Table 9 shows unrotated loadings. Most of the time, unrotated loadings are rotated in order to achieve the simplest possible factor structure, varimax rotation was used.

With varimax rotation, each factor tends to load high on a lesser number of variables and load low or very low on the other variables. For this study, loadings which are .40 or above were used so that an item could share at least 15% of its variance with the factor it loaded on. Accordingly, item 5 did not meet the criteria. In this varimax rotation (Table 10), items 1, 2, and 7 loaded on factor 1 and items 3, 4, 6 and 8 loaded on factor 2. Factor 1 was named 'integrative motivation' and factor 2 was named 'instrumental motivation,' based on the common ideas each item shares with other items. As such, loadings are used to name factors. They are Pearson product-moment correlations between each item (variable) and each of factors that is retained.

Table 9. Factor Matrix (Not Rotated)

Item No	Factor 1	Factor 2
1	.738	.0
2	.693	−.186
3	.507	.140
4	.461	.511
5	.304	.183
6	.620	.268
7	.774	−.290
8	.500	.214

Table 10. Rotated Factor Matrix

Item No	Factor 1	Factor 2
1	.640	.373
2	.676	.239
3	.339	.403
4	0	.682
5	.147	.323
6	.359	.572
7	.802	.199
8	.290	.460

After identifying common factors, one may stop here and report the dimensions of ideas he or she has found. Still, one may do further analyses depending on his or her research design. In the present research, the researcher was interested in finding out whether the gender difference existed by studying factor scores. It is only logical that, since factors have been derived, one may use factors for other analyses.

First, factor scores are calculated by the combination of factor weights and observed variables or items. SPSS programs provide factor scores for each subject based on their responses to observed items. The descriptive statistics of factor scores for the two groups are shown in Table 11. The mean scores of factor 2 for both groups of males and females were zero; for all factors, the factor scores have mean of 0 and variance of 1. That is, factor scores are standardized scores.

Table 11. Descriptive Statistics Factor Scores

	Sex	Number	Mean	Std. Deviation
Factor 1	Male	67	.15	.97
	Female	48	-.22	.62
Factor 2	Male	67	0	.85
	Female	48	0	.70

Factor 1 scores (Integrative motivation) were tested for any group difference. The t-test revealed that male subjects had higher factor scores on integrative motivation than female subjects ($t= 2.371$, $p < .05$). Factor 2 scores on instrumental motivation, however, did not differ between the groups ($t= .963$, $p> .05$).

4. Other Important Issues

In interpreting common factor analysis, it was said earlier that loadings are used. Loadings are Pearson product-moment correlations between each variable in the original set and each of common factors is retained. Tabachnick & Fidell (1996) recommends that loadings of .32 (absolute value) or above be used to specify variables that load on each component. However, Stevens (1996) proposes that we double the critical values in the following table (Table 12) for testing the significance of a loading. For example, for a sample size of 100, only loadings equal to or greater than 2x .256 = .512 in absolute value will be statistically significant. The number .256 was obtained from the table 12 because the sample size of the present study was only 115. In the present study, loadings .40 or above were used so that an item could share at least 15% of its variance with the factor it loaded on according to the criteria set by Tabachnick & Fidell (1996).

Table 12. Critical Values for Correlation Coefficient

n	Critical Value	n	Critical Value
50	.361	250	.163
80	.286	300	.149
100	.256	400	.129
140	.217	600	.105
180	.192	800	.091
200	.182	1000	.081

alpha= .01; two-tail test

Common factor analysis is different from component factor analysis in terms of total, common, and error variance. Factors resulting from common factor analysis are based only on the common variance, while components from component factor analysis are based on the total variance.

In terms of sample size, Gorsuch (1983) suggested that absolute minimum ratio should be five subjects to every variable but he added that any factor analysis should have more than 100 subjects.

In addition, although in the present study, factor scores were used for the subsequent analysis, it is possible to calculate factor-based scores. Factor-based scores can be built by summing all the variables with substantial loadings and ignoring the remaining variables with minor loadings. So, it is a sort of index construction. In the present study, item 5 did not load on any factor. Accordingly, factor-based scores might have been more meaningful than factor scores in comparing the two groups.

In terms of reporting results of factor analysis, the following are usually reported: sample size, means and standard deviations of variables in the observed variable set, correlation matrix, rotated factor loading matrix, final communities for variables in the observed variable set, and for each rotated factor, name of factor, sum of squares of loadings, and proportion of total and common variance explained.

5. Conclusion

With the application of common factor analysis, the present study found that the subjects held two kinds of motivation, integrative motivation and instrumental motivation. The finding corroborated the hypotheses of Gardner(1989). In addition, it was found that male subjects had higher factor scores on integrative motivation than female subjects had. In addition, some of processes were reviewed in the study. As stated earlier, common factor analysis is a useful statistical method to investigate dimensions of ideas. It can be applied to simply make an instrument that measures psychological constructs. Also, it can be employed to investigate further questions by calculating factor scores or factor-based scores.

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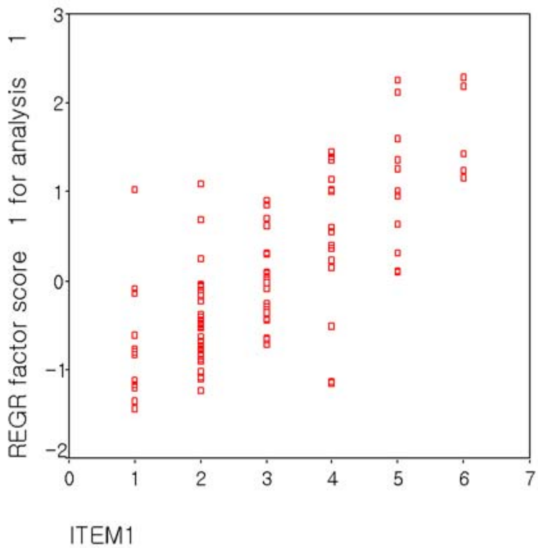
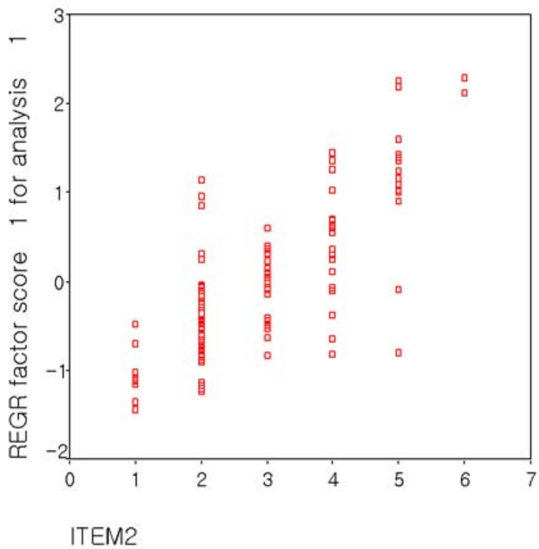
Appendix A. Graphs of Correlations Between Factors and Items

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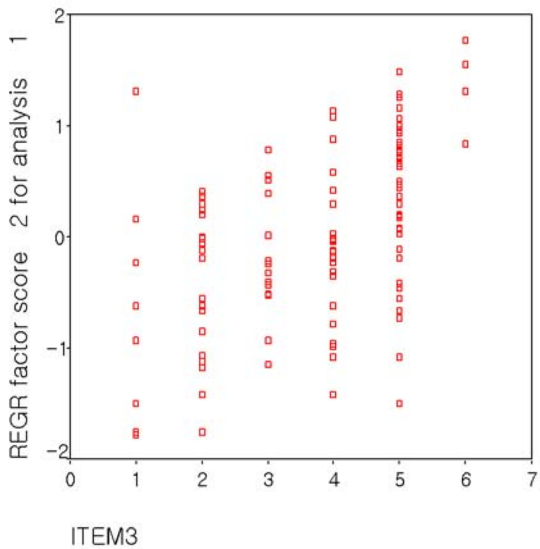
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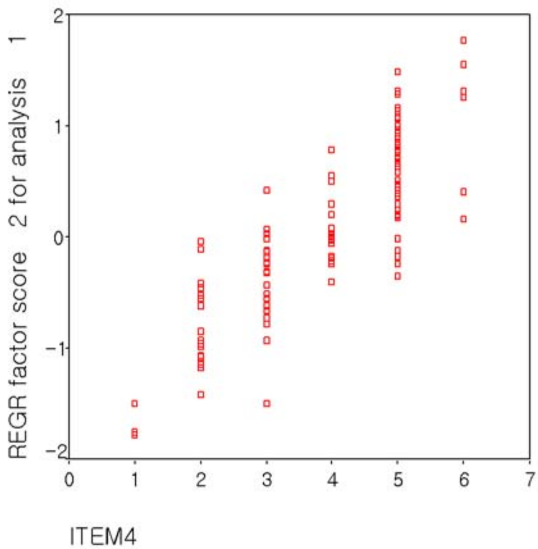
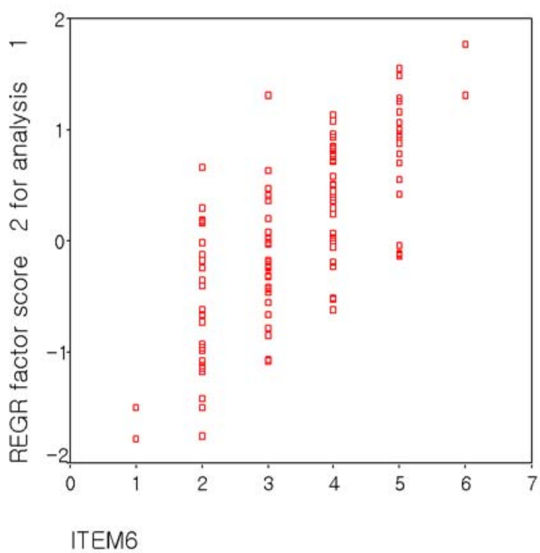
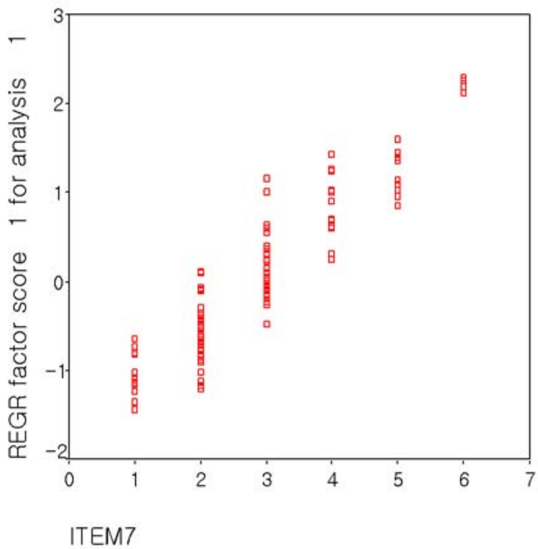




* A graph for item 5 was deleted because the item did not load onto any factor.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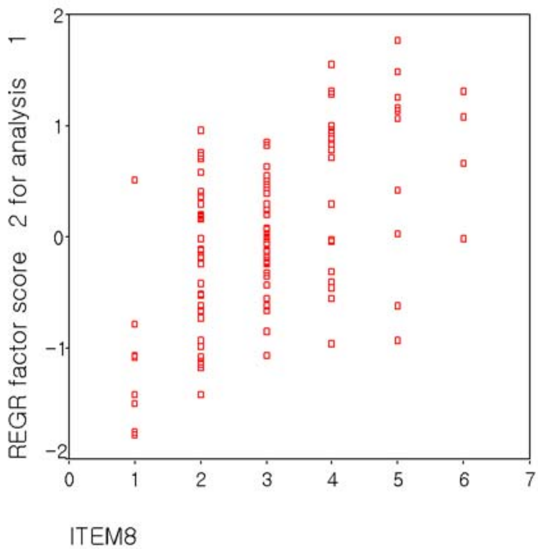
ITEM7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ITEM8

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